Calcul intégral $2^{ème}$ Bac Pro

Correction des exercices du polycopié du cours

EX.1 Vérifier les calculs suivants:

1°/ $\int_{-1}^{0}(2x+1)\,dx = 0$

2°/ $\int_{0}^{1}(x^3+6x+1)\,dx = \dfrac{13}{4}$

3°/ $\int_{4}^{16}\dfrac{dx}{2\sqrt{x}} = 2$; 4°/ $\int_{0}^{3}\dfrac{dx}{x+2} = \ln\left(\dfrac{5}{2}\right)$

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solution:

1°/ $\int_{-1}^{0}(2x+1)\,dx \overset{?}{=} 0$

on a: $\int_{-1}^{0}(2x+1)\,dx = \int_{-1}^{0}f(x)\,dx$

avec: $f(x) = 2x+1$

une primitive de f: $F(x) = x^2+x$

car: $F'(x) = (x^2+x)' = 2x+1$

donc: $\int_{-1}^{0}(2x+1)\,dx = \int_{-1}^{0}F'(x)\,dx$

$= \left[F(x)\right]_{-1}^{0} = \left[x^2+x\right]_{-1}^{0}$

$= 0^2+0 - \left((-1)^2+(-1)\right) = -(1-1) = \boxed{0}$

2°/ $\int_{0}^{1}(\underbrace{x^2+6x+1}_{f(x)})\,dx \overset{?}{=} \dfrac{13}{4}$

$\Rightarrow F(x) = \dfrac{x^3}{3}+6\dfrac{x^2}{2}+x$; F est une primitive.

donc:

$\int_{0}^{1}f(x)\,dx = \left[F(x)\right]_{0}^{1} = F(1)-F(0)$

avec: $\begin{cases} F(1) = \dfrac{1}{3}+\dfrac{6}{2}+1 = \dfrac{1}{3}+4 = \dfrac{13}{3} \\ F(0) = 0+0+0 = 0 \end{cases}$

donc: $\int_{0}^{1}(x^2+6x+1)\,dx = \boxed{\dfrac{13}{3}}$

3°/ $\int_{4}^{16}\dfrac{dx}{2\sqrt{x}} = \int_{4}^{16}\underbrace{\dfrac{1}{2\sqrt{x}}}_{f(x)}\,dx \overset{?}{=} 2$

$f(x) = \dfrac{1}{2\sqrt{x}} \Rightarrow F(x) = \sqrt{x}$

donc: $\int_{4}^{16}f(x)\,dx = \left[\sqrt{x}\right]_{4}^{16} = \sqrt{16}-\sqrt{4}$

$= 4-2 = \boxed{2}$

4°/ $\int_{0}^{3}\dfrac{dx}{x+2} = \int_{0}^{3}\dfrac{1}{x+2}\,dx \overset{?}{=} \ln\left(\dfrac{5}{2}\right)$

$f(x) = \dfrac{1}{x+2} = \dfrac{(x+2)'}{x+2} = \ln'(x+2)$

$F(x) = \ln(x+2)$ donc:

$\int_{0}^{3}\dfrac{dx}{x+2} = F(x)]_{0}^{3} = \left[\ln(x+2)\right]_{0}^{3}$

$= \ln(3+2)-\ln(0+2) = \ln 5 - \ln 2$

$= \boxed{\ln\left(\dfrac{5}{2}\right)}$

EX.2 Montrer que:

1°/ $\int_{0}^{\pi}\left(4x+\dfrac{2}{3}\sin(x)\right)\,dx = 2\pi^2+\dfrac{4}{3}$

2°/ $\int_{0}^{1}(5x^3+e^x)\,dx = e+\dfrac{1}{4}$

3°/ $\int_{1}^{e}\left(\dfrac{1}{x}-\dfrac{1}{x^2}\right)\,dx = \dfrac{1}{e}$

4°/ $\int_{1}^{2}(2x+3)(x^2+3x)^2\,dx = 312$

Solution:

1°/ $\int_{0}^{\pi}\left(4x+\dfrac{2}{3}\sin(x)\right)\,dx$

$= 4\times\int_{0}^{\pi}x\,dx + \dfrac{2}{3}\int_{0}^{\pi}\sin(x)\,dx$

$= 4\left[\dfrac{x^2}{2}\right]_{0}^{\pi} + \dfrac{2}{3}\left[-\cos(x)\right]_{0}^{\pi}$

$= 4\left[\dfrac{\pi^2}{2}-0\right] + \dfrac{2}{3}\left[-(-1)+1\right]$

$= 4\times\dfrac{\pi^2}{2} + \dfrac{2}{3}\times 2 = \boxed{2\pi^2+\dfrac{4}{3}}$

2°/ $\int_{0}^{1}(5x^3+e^x)\,dx = 5\times\int_{0}^{1}x^3\,dx+\int_{0}^{1}e^x\,dx$

$= 5\left[\dfrac{x^4}{4}\right]_{0}^{1} + \left[e^x\right]_{0}^{1} = 5\times\left(\dfrac{1}{4}-0\right)+e^1-e^0$

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①

$$\int_0^1 (5x^3 + e^x)\,dx = \frac{5}{4} + e - 1$$

$$= e + \frac{5}{4} - 1 = e + \frac{1}{4}$$

3°/ $\int_1^e \frac{1}{x} - \frac{1}{x^2}\,dx = \int_1^e \frac{1}{x}\,dx + \int_1^e -\frac{1}{x^2}\,dx$

$$= \left[\ln(x)\right]_1^e + \left[\frac{1}{x}\right]_1^e$$

$$= \ln(e) - \ln(1) + \frac{1}{e} - \frac{1}{1}$$

$$= 1 - 0 + \frac{1}{e} - 1 = \boxed{\frac{1}{e}}$$

car: $\ln'(x) = \frac{1}{x}$ et $\left(\frac{1}{x}\right)' = -\frac{1}{x^2}$

donc: $\frac{1}{x} - \frac{1}{x^2} = \left(\ln(x) + \frac{1}{x}\right)'$

4°/ $\int_1^2 \underbrace{(2x+3)(x^2+3x)^2}_{f(x)}\,dx$

$f(x) = (x^2+3x)'(x^2+3x)^2$

$$= \frac{1}{3} \times \left[3(x^2+3x)'(x^2+3x)^2\right]$$

$$= \frac{1}{3} F'(x) \; ; \; \text{avec: } F(x) = (x^2+3x)^3$$

donc: $\int_1^2 f(x)\,dx = \frac{1}{3}\left[F(x)\right]_1^2 = \frac{1}{3}(F(2)-F(1))$

$\begin{cases} F(2) = (2^2+3\times2)^3 = (4+6)^3 = 1000 \\ F(1) = (1+3)^3 = 4^3 = 64 \end{cases}$

$\int_1^2 f(x)\,dx = \frac{1}{3}\left[1000 - 64\right] = \frac{936}{3} = \boxed{312}$

Solution: 1°/ $\int_0^3 2x\,e^{x^2}\,dx$

$f(x) = (x^2)'\,e^{x^2} = \left(e^{x^2}\right)' = F'(x)$

avec: $F(x) = e^{x^2}$ donc:

$\int_0^3 f(x)\,dx = \left[F(x)\right]_0^3 = F(3) - F(0)$

$$= e^{3^2} - e^{0^2} = e^9 - e^0 = \boxed{e^9 - 1}$$

2°/ $\int_1^2 \sqrt{x}\,dx = \int_1^2 x^{1/2}\,dx$

$$= \left[\frac{x^{\frac{1}{2}+1}}{\frac{1}{2}+1}\right]_1^2 = \left[\frac{x^{\frac{1}{2}+1}}{3/2}\right]_1^2$$

$$= \frac{2^{\frac{1}{2}+1}}{3/2} - \frac{1}{3/2} = \frac{2^{1/2} \times 2^1}{3/2} - \frac{2}{3}$$

$$= \sqrt{2} \times 2 \times \frac{2}{3} - \frac{2}{3} = \boxed{\frac{4}{3}\sqrt{2} - \frac{2}{3}}$$

3°/ $\int_1^2 \frac{2x+3}{x^2+3x}\,dx = \int_1^2 \frac{(x^2+3x)'}{x^2+3x}\,dx$

$$= \int_1^2 \ln'(x^2+3x)\,dx = \left[\ln(x^2+3x)\right]_1^2$$

$$= \ln(2^2+3\times2) - \ln(1+3\times1)$$

$$= \ln(10) - \ln(4) = \ln\left(\frac{10}{4}\right) = \boxed{\ln\left(\frac{5}{2}\right)}$$

Solution:

1°/ $\int_{-1}^2 x\,e^x\,dx = \int_{-1}^2 u(x)\,v'(x)\,dx$

avec: $\begin{cases} u(x) = x \\ v'(x) = e^x \end{cases}$ donc: $\begin{cases} u'(x) = 1 \\ v(x) = e^x \end{cases}$

②

par suite:
$$\int u(x)v'(x)\,dx = \big[u(x)v(x)\big] - \int u'(x)\,v(x)\,dx$$

$$\Rightarrow \int_{-1}^{2} xe^{x}\,dx = \big[1x\,e^{x}\big]_{-1}^{2} - \int_{-1}^{2} 1x\,e^{x}\,dx$$

$$= e^{2} - \bar{e}^{1} - \int_{-1}^{2} e^{x}\,dx$$

$$= e^{2} - \bar{e}^{1} - \big[e^{x}\big]_{-1}^{2} = e^{2} \div \bar{e}^{1} - (e^{2} - \bar{e}^{1})$$

$$= e^{2} - \bar{e}^{1} - e^{2} + \bar{e}^{1} = \boxed{0}$$

2°/ $\displaystyle\int_{0}^{\pi} x\sin(x)\,dx = \int_{0}^{\pi} \underbrace{x}_{u(x)}\times \underbrace{\sin(x)}_{v'(x)}\,dx$

$\begin{cases} u(x)=x \\ v'(x)=\sin(x) \end{cases} \Rightarrow \begin{cases} u'(x)=1 \\ v(x)=-\cos(x) \end{cases}$

donc
$$\int u(x)\,v'(x)\,dx = \big[u(x)v(x)\big] - \int u'(x)v(x)\,dx$$

$$\int_{0}^{\pi} x\sin(x)\,dx = \big[-x\cos(x)\big]_{0}^{\pi} - \int_{0}^{\pi} 1\times(-\cos(x))\,dx$$

$$= \big[-\pi(-1)+0\big] + \int_{0}^{\pi} \cos(x)\,dx$$

$$= \pi + \big[\sin(x)\big]_{0}^{\pi} = \pi + (0-0) = \boxed{\pi}$$

3°/ $\displaystyle\int_{2}^{e} 4x^{3}\ln(x)\,dx = \int_{2}^{e} (4x^{3})\,\ln(x)\,dx$

$\begin{cases} u'(x)=4x^{3} \\ v(x)=\ln(x) \end{cases} \Rightarrow \begin{cases} u(x)=x^{4} \\ v'(x)=\dfrac{1}{x} \end{cases}$

$$\int_{2}^{e} 4x^{3}\ln(x)\,dx = \big[u(x)v(x)\big]_{2}^{e} - \int_{2}^{e} u(x)\,v'(x)\,dx$$

$$= \big[x^{4}\ln(x)\big]_{2}^{e} - \int_{2}^{e} x^{4}\times\frac{1}{x}\,dx$$

$$= e^{4}\ln(e) - 2^{4}\ln(2) - \int_{2}^{e} x^{3}\,dx$$

$$= e^{4}\times 1 - 16\ln(2) - \Big[\frac{x^{4}}{4}\Big]_{2}^{e} \quad ③$$

$$= e^{4} - 16\ln(2) - \Big(\frac{e^{4}}{4} - \frac{2^{4}}{4}\Big)$$

$$= e^{4} - \frac{e^{4}}{4} - 16\ln(2) + \frac{16}{4}$$

$$= \Big(1-\frac{1}{4}\Big)e^{4} - 16\ln(2) + 4$$

$$= \boxed{\frac{3}{4}e^{4} - 16\ln(2) + 4}$$